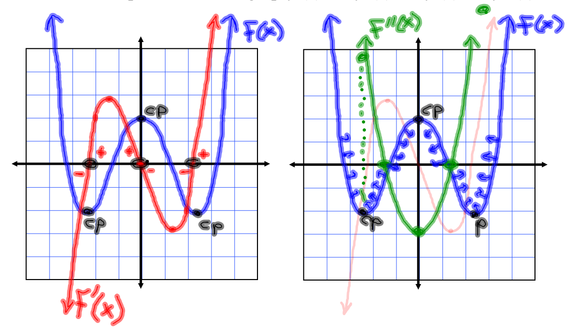
Calculus AB

3-3 and 3-4

1st and 2nd Derivative Tests

Given: $f(x) = \frac{1}{8}x^4 - \frac{3}{2}x^2 + 2$

Look for relationships between the graph $f(x)$ and $f'(x)$ and $f(x)$ and $f''(x)$.



First Derivative Test- Let c be a critical number of a function f that is continuous on an open interval (a,b), containing c. If f is differentiable on the interval, except possibly at c, then:

1) if $f'(x)$ changes from negative to positive at c, then $f(c)$ is:
a minimum.

2) if $f'(x)$ changes from positive to negative at c, then $f(c)$ is:
a maximum

3) if $f'(x)$ does not change signs at c, then $f(c)$ is:
test fails.
(could be a point of inflection)

Second Derivative Test- Let f be a function with critical point c such that $f'(c) = 0$ and $f''(x)$ exists on an open interval (a,b), containing c.

1) if $f''(x) > 0$, then $f(c)$ is: a minimum.
f is concave up.

2) if $f''(x) < 0$, then $f(c)$ is: a maximum.
f is concave down.

3) if $f''(x) = 0$, then the test fails (use first derivative test).

Point of Inflection- point where concavity changes, always has $f''(x)=0$ or f'' undefined

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Find the critical points of f (if any). Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. (pg 186)

22) $f(x) = x^3 - 6x^2 + 15$

Find c.p. ($f'(x)=0$ or undefined)

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

Extrema (either do 1st derivative test or second)

1st

$f'(-1) = 15$ +
$f'(0) = 0$
$f'(1) = -9$ −
max $x=0$

$f'(3) = -9$
$f'(4) = 0$ − to +
$f'(5) = 15$
min at $x=4$

increasing $(-\infty, 0) \cup (4, \infty)$
decreasing $(0, 4)$

2nd

$f''(x) = 6x - 12$

$f''(0) = -12$ $<0 \rightarrow$ max

$f''(4) = 12$ $>0 \rightarrow$ min

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36) $f(x) = \dfrac{x+4}{x^2}$

c.p.
$-8, 0$

$f'(x) = \dfrac{1(x^2) - 2x(x+4)}{x^4} = \dfrac{1x^2 - 2x^2 - 8x}{x^4}$

$= \dfrac{-x^2 - 8x}{x^4} = \dfrac{-x(x+8)}{x^4} = \dfrac{-(x+8)}{x^3}$

$0 = -(x+8)$, $x = -8$, undefined at $x = 0$

extrema

1st

$f'(-1) = 7$
$f'(0) = \varnothing$
$f'(1) = -9$
Looks like max, f is undefined at 0

$f'(-10) = -$
$f'(-8) = 0$
$f'(-1) = 7$
min at $x=-8$

we are really only interested in whether this value is positive or negative.

decrease : $(-\infty, -8) \cup (0, \infty)$
increase : $(-8, 0)$

Assignment:

Pg. 186

17-41 odd,
85

Find all points of inflection and discuss the concavity of the function. (pg 195)

20) $f(x) = -x^4 + 24x^2$

$F'(x) = -4x^3 + 48x$

$F''(x) = -12x^2 + 48$

$0 = -12(x^2 - 4)$

$0 = -12(x+2)(x-2)$

$x = 2,\ x = -2$

$F''(1) = 36$

$F''(2) = 0$

$F''(3) = -60$

$F''(-3) = -60$

$F''(-2) = 0$

$F''(-1) = 36$

$(2, 80)$

$(-2, 80)$

concave up
$(-2, 2)$

concave down
$(-\infty, -2) \cup (2, \infty)$

In this Second Derivative Test, we need a value on either side of the critical point to determine whether it is concave up or down.

Find all relative extrema.

40) $f(x) = x^2 + 3x - 8$

$\left(-\frac{3}{2}, -\frac{41}{4}\right)$

Find c.p.

$F'(x) = 2x + 3$

$0 = 2x + 3$

$x = -\frac{3}{2}$

2nd deriv. test

$F''(x) = 2$

$F''\left(-\frac{3}{2}\right) = 2 > 0$

min.

Assignment 2:

Pg. 195
19-51 odd
69, 77, 79